MULTIMEDIA UNIVERSITY

STUDENT ID NO

MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 1, 2022/2023 (TERM ID 2210 & TERM ID 2215)

PEM0036 – CALCULUS

(Foundation in Engineering)

17 FEBRUARY 2023
9.00 a.m. – 11.00 a.m.
(2 Hours)

INSTRUCTIONS TO STUDENT

1. This question paper consists of **SIX** printed pages including the cover page and appendix.

2. Attempt **ALL FOUR** questions. All questions carry equal marks and the distribution of the marks for each question is given.

3. Please write all your answers in the answer booklet provided. All necessary working steps MUST be shown.

APPENDIX

BASIC DIFFERENTIATION AND INTEGRATION FORMULAS

$$\frac{d}{dx}[\sin x]=\cos x \qquad \frac{d}{dx}[\cos x]=-\sin x$$

$$\frac{d}{dx}[\tan x]=\sec^2 x \qquad \frac{d}{dx}[\cot x]=-\csc^2 x$$

$$\frac{d}{dx}[\sec x]=\sec x\tan x \qquad \frac{d}{dx}[\csc x]=-\csc x\cot x$$

$$\frac{d}{dx}[e^x]=e^x \qquad \frac{d}{dx}[\ln x]=\frac{1}{x};\quad x>1$$

$$\frac{d}{dx}[\sin^{-1}x]=\frac{1}{\sqrt{1-x^2}};\quad |x|<1 \qquad \frac{d}{dx}[\cos^{-1}x]=-\frac{1}{\sqrt{1-x^2}};\quad |x|<1$$

$$\frac{d}{dx}[\tan^{-1}x]=\frac{1}{1+x^2} \qquad \frac{d}{dx}[\cot^{-1}x]=-\frac{1}{1+x^2}$$

$$\frac{d}{dx}[\sec^{-1}x]=\frac{1}{|x|\sqrt{x^2-1}};\quad |x|>1 \qquad \frac{d}{dx}[\csc^{-1}x]=-\frac{1}{|x|\sqrt{x^2-1}};\quad |x|>1$$

$$Area=\int_a^b [f(x)-g(x)]\ dx$$

$$Volume\ (Disk)=\pi\int_a^b [f(x)]^2\ dx$$

$$Volume\ (Washer)=\pi\int_a^b [f(x)]^2-[g(x)]^2\ dx$$

$$Volume\ (Cylindrical\ Shells)=\int_a^b 2\pi(shell\ radius)(shell\ height)\ dx$$

Continued...

Answer ALL questions [100 marks].

QUESTION 1 [25 marks]

a. Find each limit if it exits.

i. $\lim_{x\to 3^-} \frac{1}{(x-3)^3}$ [1.5 marks]

ii. $\lim_{x\to 0} \left(\frac{\sin 3x}{3x} + \frac{1}{e^x}\right)$ [2.5 marks]

b. If $f(x) = \frac{\sqrt{4x^2+9}}{3x+2}$, find

i. $\lim_{x\to 2} f(x)$. [2 marks]

ii. $\lim_{x\to \infty} f(x)$. [4 marks]

iii. $\lim_{x\to -\infty} f(x)$. [4 marks]

iv. the horizontal or/and vertical asymptote of the function $f(x)$. [4 marks]

c. A function f is defined by

$$f(x) = \begin{cases} 2e^{ax} - 5 & ; \quad x \leq 0 \\ x - b & ; \quad 0 < x < 2 \\ x^3 - 10 + a & ; \quad x \geq 2 \end{cases}$$

i. If $\lim_{x\to 0} f(x)$ exist and $\lim_{x\to 2} f(x) = -1$,
find the constant values of a and b. [5 marks]

ii. Determine whether $f(x)$ is continuous at $x = 2$? [2 marks]

Continued...

QUESTION 2 [25 marks]

a. Find $\frac{dy}{dx}$ for $y = \ln(e^x) - \cos^2 x$. [2 marks]

b. If $y = \int_0^x (2x - 1)\, dx$, determine $\frac{dy}{dx}$ at $x = \frac{1}{2}$. [2 marks]

c. Given $y = e^{-2x} \cdot \sin x$, show that $y'' + 4y' + 5y = 0$. [6 marks]

d. For the function $f(x) = x^3 - 3x - 2$, find the following:

i. Intervals on which the function is increasing or decreasing. [6 marks]

ii. Extreme values(s) and where they occur. [2 marks]

iii. Inflection point(s). [4 marks]

iv. Hence, sketch the graph of $f(x)$. [3 marks]

Continued ...

QUESTION 3 [25 marks]

a. Evaluate $\int_0^4 (2x - \frac{1}{\sqrt{x}})\, dx$. [2.5 marks]

b. Show that $\int \frac{1}{x^2-b^2}\, dx = \frac{1}{2b} \ln \left|\frac{x-b}{x+b}\right| + c$, where b and c are constant.

Hence, find $\int \frac{1}{x^2-25}\, dx$. [9.5 marks]

[Hint: Integration by partial fractions]

c. Figure 1 shows a shaded region $\boldsymbol{R}$ bounded by the curves

$y = \frac{6}{x} - 1$ and $y = 6x - x^2$.

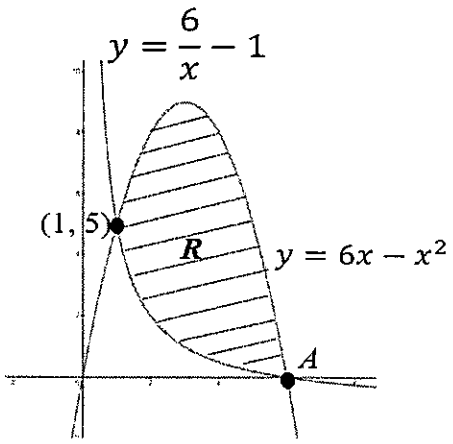

Figure 1

i. Find the point $\boldsymbol{A}$. [2.5 marks]

ii. Hence, find the area of the region bounded by the two curves. [4 marks]

iii. Use volume by shell method to evaluate the volume if the region $\boldsymbol{R}$ is revolved about the line $x = 6$. [6.5 marks]

Continued ...

QUESTION 4 [25 marks]

a. Given that $y = 0$ when $x = 2$, find the particular solution of the differential equation $\frac{dy}{dx} = \frac{x}{e^y}$, expressing y in terms of x. [6 marks]

b. Solve the first order differential equation:

$$x\frac{dy}{dx} + 2y = 2x + 3$$ [6 marks]

c. Solve $y'' - 4y' - 5y = -2e^{3x}$ if $y(0) = \frac{1}{4}$ and $y'(0) = \frac{27}{4}$. [13 marks]

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